Introduction to scalar quantization[1]

Mu Li

iPAL Group Meeting

November 12, 2010

[1] Gersho and Gray, vector quantization and signal compression

Outline

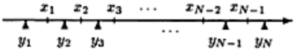
- Related definitions of quantizer
- Quantizer structure
- Measuring quantizer performance
- Uniform quantizer
- Nonuniform quantizer and compandor

Related definitions of quantizer

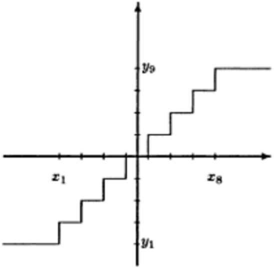
1. Quantization is the heart of analog-to-digital conversion.
2. An N-point scalar quantizer Q is a mapping: $\mathcal{R} \mapsto \mathcal{C}$ where $\mathcal{R}$: real line
   $\mathcal{C} = \{y_1, y_2, \ldots, y_N\} \subset \mathcal{R}$
   y_i: output values, $\{C\}$: $codebook$
3. Usually assuming $y_1 < y_2 < \ldots, < y_N$
4. Resolution: number of bits needed to uniquely specify the quantized value. $r = log_2 N$
5. Every N point quantizer corresponds to a partition of the real line $\mathcal{R}$ into N cells, $\{R_1, R_2, \ldots, R_N\}$, where $R_i = \{x \in \mathcal{R} : Q(x) = y_i\}$
6. Overload cell: a cell that is unbounded.
   Granular cell: a cell that is bounded.
7. A quantizer is completed described by output values $\{y_i, i = 1 \ldots N\}$ and partition cells $\{R_i, i = 1 \ldots N\}$

Related definitions of quantizer

1. Regular quantizer: each cell R_i is an interval and $y_i \in (x_{i-1}, x_i)$



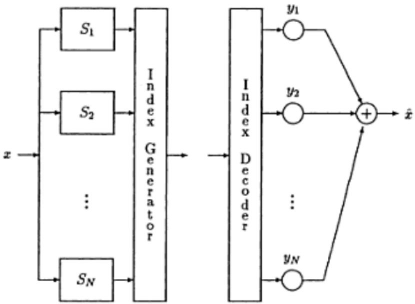
2. A typical graph of staircase character of a quantizer:



3. Each quantizer can be viewed as a combination of an encoder, ξ, and a decoder, $\mathcal{D}$. If $Q(x) = y_i$, then $\xi(x) = i, \mathcal{D}(i) = y_i$

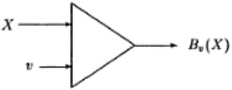
Primary quantizer structure

1. Indicator function $1_R(x) = 1$ if $x \in R; 0$ o.w.
2. Selector function $S_i(x) = 1_{R_i}(x)$
3. Quantizer $Q(x) = \sum_{i=1}^{N} y_i S_i(x)$
4. Primary structure:

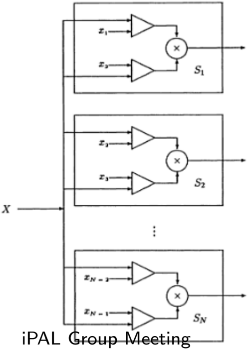


Secondary quantizer structure

1. Indicator function $B_v(x) = 1_{(-\infty,v]}(x) = 1$ if $x \leq v; 0$ o.w.
   Comparator:

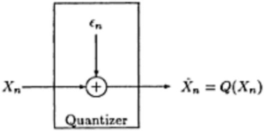


2. Selector $S_i(x) = B_{x_i}(x)\overline{B}_{x_{i-1}}(x)$
3. Secondary structure:



Measuring quantizer performance

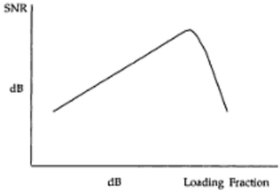
1. Average distortion
$D = E[(X - Q(X))^2] = \sum_{i=1}^{N} \int_{R_i} (x - y_i)^2 f_X(x) dx$
2. Signal-to-Noise Ratio: $SNR = 10 \lg_{10} E(X^2)/D$
3. Additive noise model:



4. Granular noise: noise caused by bounded cells;
overload noise: noise caused by unbounded cells.
5. Loading factor: $\gamma = V/\sigma$, where $V = x_{N-1}, \sigma$ is signal standard deviation.

Uniform quantizer

- Definition: $y_i - y_{i-1} = \Delta, i = 1, \ldots, N$ and $y_i = (x_{i-1} + x_i)/2, i = 2, \ldots, N-1$
- Average distortion: $D = \Delta^2/12$
- SNR vs. loading fraction, which is the reciprocal of loading factor.

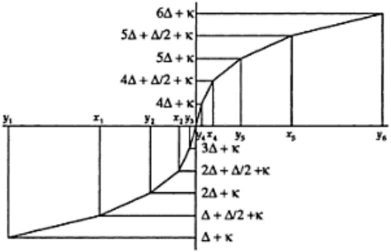


Nonuniform quantizer and compandor

- The spacing of quantization levels are nonuniform.
- Comparing with uniform quantizer, the dynamic range can be increased.
- Nonuniform quantizer has a general model: compandor.

Nonuniform quantizer and compander

- Informal proof:
  $P = \{(y_i, i\Delta + k), i = 1, \ldots, N\}$
  $Q = \{(x_i, i\Delta + \Delta/2 + k), i = 1, \ldots, N-1\}$
  $G(x_i) = i\Delta + \Delta/2 + k$
  $G(y_i) = i\Delta + k$



Conclusion

- Quantization is the heart of analog-to-digital conversion, which is usually non-linear and doesn't have analytical expression.
- Quantizer can have structural representation which is based on selector or comparator.
- Average distortion, SNR and loading factor are three important parameters measuring a quantizer's performance.
- The uniform quantizer is the optimal quantizer that minimizes the average distortion when the input is uniformly distributed.
- Nonuniform quantizer has a general compandor model, which can have wider dynamic range than uniform quantizers.